Improved Trajectory Planning for On-Road Self-Driving Vehicles via Combined Graph Search, Optimization & Topology Analysis

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Agenda

- Introduction / Related Work / Motivation
- Proposed Planning Framework
  - Approach
    - Proposed Method I: Region Segmentation w/ Topology Analysis
    - Proposed Method II: Search-based Planning w/ Edge-augmented Graph
    - Adapted Method I: LQR Path/Trajectory Smoothing
    - Proposed Method III: Sampling-based Maneuver Pattern Analysis
    - Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR
- Results
- Conclusion/Contribution/Future Work
Introduction

• Self-driving / Autonomous-driving Vehicles

   Boss          Cadillac          Google          Uber


• Social Benefits

   Safety

   Two million car crashes per year in the United States [1].

   Good Autonomy Software!

   Wasted hours in traffic jams.
   Wasted parking space to city.

   Autonomous Driving

   Efficiency

   Freedom

   Driving is exhausting.
   Driving is difficult/impossible for some.

Introduction

• Thesis Scope

Commonly used architecture (at least conceptually):
1. Urban challenge entries.
2. Main self-driving players.

Related Work

• Sampling-based Planners

• Optimization-based Planners

• Hybrid Sampling/Optimization Planners

• Topology-aware Planners
Related Work

- Sampling-based Planners

1. Lightweight primitive sampling with model-based evaluation.
2. Deterministic runtime with termination guarantee.
3. Search space is comprehensive with respect to the continuous space of interest.

Pro

Con

1. Sampling sub-optimality, not converging to local minimum.
2. Search space blow-up due to the curse of dimensionality.
Related Work

- Optimization-based Planners

**Direct Method**

- Search in the continuous space, alleviate sampling sub-optimality.
- Efficient if implemented properly, compared to the thousands, even millions of sampled trajectory evaluations.

**Indirect Method**

- Lacks global awareness, can get stuck in the wrong local minimum.
- Non-deterministic runtime w/o termination guarantee
- Difficult to parallelize the computation.

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[Rösmann et al., 2012]

[Thrun et al., 2006]

[van den Berg, 2016]
Related Work

- Hybrid Sampling/Optimization Planners

**Hybrid A* + Conjugate Gradient Optimization**

- Achieve local minimum while having global awareness.
- Moderate computation overhead.

**Lattice Sampling + Simplex Optimization**

- Pure path/speed planning, independent spatial/temporal planning.
- Direct trajectory optimization by manipulating the sampled configuration states. Difficult to guarantee plan feasibility.
Related Work

- Topology-aware Motion Planners

**Visibility Graph & Voronoi Graph**

- [Schmitzberger, 2002] & [Choset, 2005]

**Cell Decomposition + Mixed-Integer-Quadratic Programming**

- [Parker, 2016]

**Homology Marker Function**

- [Bhattacharya, 2012]

**Pro**

1. Explicit topological awareness.
2. Focus (narrow down) the search space.

**Con**

1. Pure spatial topological analysis.
2. Co-terminal requirements for paths under analysis.
Motivation

• Thesis Statement:

*By taking advantage of a combined sampling-n-search, optimization and topology analysis approach, we can avoid the pitfalls of standalone methods, and equip self-driving cars with improved high-level reasoning capabilities for on-road trajectory planning.*

**Requirement 1:** a deliberative trajectory planning system.

**Requirement 2:** spatiotemporal (trajectory) planning.

**Requirement 3:** tactical reasoning capability with topological awareness.

**Requirement 4:** global awareness with the ability to converge to a local optimum.

**Requirement 5:** apply to self-driving passenger vehicle on-road driving.
Proposed Planning Framework

- Technical Overview

**Sampling & Search/Topology-based Planning**
- Spatial Graph Segmentation based on Topology Analysis
- Reference Path Planning with Edge-augmented Graph Search
- Trajectory Sampling-based Maneuver Pattern Analysis

**Optimization-based Planning**
- Reference Path Smoothing with LQR Controller
- Reference Speed Profile Generation
- Focused Trajectory Refinement with Iterative-LQR Optimizer

**High-level Tactical Reasoning Capability**

By taking advantage of a combined sampling-n-search, optimization and topology analysis approach, we can avoid pitfalls of standalone methods, and equip self-driving cars with improved high-level reasoning capabilities for on-road trajectory planning.
Proposed Method I: Region Segmentation w/ Topology Analysis
Proposed Method I: Region Segmentation w/ Topology Analysis

- Homology Marker Function-based Topological Analysis

**Biot-Savart Law:**
A steady current flowing through a wire generates a magnetic field $\mathbf{B}$.

$$B(r) = \frac{\mu_0 I}{4\pi} \int \frac{dl \times (l-r)}{||l-r||^3}$$

**Homology Marker Function**

$$\mathcal{H}(\mathcal{T}) = \int_{\mathcal{T}} \mathbf{B} \cdot dl$$

$\mathcal{H}(\mathcal{T}_1) = \mathcal{H}(\mathcal{T}_3) \neq \mathcal{H}(\mathcal{T}_2)$

- $\mathcal{T}_1$ & $\mathcal{T}_3$ are homological.
- $\mathcal{T}_1$ & $\mathcal{T}_2$ are NOT homological.

**Magnetic field generated to distinguish trajectories.**
Proposed Method I: Region Segmentation w/ Topology Analysis

- Segment directed acyclic graph (DAG) into several sub-graphs

No obstacle $\Rightarrow$ No segmentation

Obstacle $\Rightarrow$ Two Sub-Graphs
Proposed Method I: Region Segmentation w/ Topology Analysis

- Dynamic Programming-Inspired Backward Topology Induction

**Step 1: Construct Topology Graph**

\[ \Delta_i^H = \mathcal{H}(e_i) = \int_{e_i} B \cdot dl \]

**Step 2: Backward Topology Induction**

*Dynamic Programming*

\[
\begin{align*}
\Delta_i^H + \Delta_j^H &= \Delta_i^H + \Delta_j^H \\
\Delta_i^H + \Delta_j^H &= \Delta_i^H + \Delta_j^H
\end{align*}
\]

**Step 3: Forward Region Marking**
Proposed Method II: Search-based Planning w/ Edge-augmented Graph
Proposed Method II: Search-based Planning w/ Edge-augmented Graph

- Motivation: why do we need edge-augmented graph?

Edge-based Cost:
- What if the cost is associated with two neighboring edges?

Node-based Cost:

- Smoothing Cost
- Contractive Force in Elastic-Band

[Diagram showing edge-based and node-based costs with an example of smoothing cost and contractive force in elastic-band]
Proposed Method II: Search-based Planning w/ Edge-augmented Graph

- Construct & Search over Edge-augmented Graph
  
  1. Construct regular DAG
  
  2. Evaluate graph node/edge
  
  3. Construct edge-augmented node
  
  4. Evaluate edge-augmented node
  
  5. Build edge-augmented graph
    
    Edge-augmented graph is still DAG!
  
  6. Search DAG with dynamic programming or topological search.
Adapted Method I: LQR-based Path/Trajectory Smoothing
Adapted Method I: LQR-based Path/Trajectory Smoothing

- **Motivation:** How to convert coarse graph plan to smooth path/trajectory?

  - Spatial Piecewise-Linear (Path) Plan
  - Spatiotemporal Piecewise-Linear (Trajectory) Plan

  Roughly captures the gist of maneuver.

  Not model-feasible, non-smooth.

**Solution:**

- Treat piecewise linear plan as a coarse reference.
- Use a realistic vehicle model and a trajectory tracking controller to “follow” the reference.
- Keep the trace of the model states as the smoothed trajectory.

**What vehicle model to use?**

- Geometric tracker, e.g., pure pursuit.
- Optimal trackers, e.g., LQR-based tracker.
Adapted Method I: LQR-based Path/Trajectory Smoothing

- Path/Trajectory Smoothing

### Lateral Tracking Control

**State Transformation:**
\[
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} d \\ \theta^* - \theta \\ \delta \end{bmatrix}
\]

**Dynamics Linearization:**
\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & -\frac{v}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \delta
\]

### Longitudinal Tracking Control

**State Transformation:**
\[
x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} l \\ v^* - v \end{bmatrix}
\]

**Dynamics Linearization:**
\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} a
\]

### Linear Quadratic Regulator

**Minimize:**
\[
J = \int_0^\infty (x^T Q x + u^T R u) \, dt
\]

**Control:**
\[
u = -K \cdot x
\]

\[
K = R^{-1} B^T P
\]

\[
A^T P + PA - PBR^{-1} B^T P + Q = 0
\]

---

Smoothed model-feasible path/trajectory
Proposed Method III: Sampling-based Maneuver Pattern Analysis

1. Spatial Graph Segmentation based on Topology Analysis
2. Reference Path Planning with Edge-augmented Graph Search
3. Reference Path Smoothing with LQR Controller
4. Reference Speed Profile Generation
5. Trajectory Sampling-based Maneuver Pattern Analysis
6. Trajectory Smoothing with LQR Controller
7. Focused Trajectory Refinement with Iterative-LQR Optimizer

Reference -> Trajectory
Proposed Method III: Sampling-based Maneuver Pattern Analysis

Motivation: factors other than topology that matter for pattern distinction of spatiotemporal trajectory?

**Co-terminal**

Homology Marker Function
\[ \mathcal{H}(\mathcal{T}) = \int_{\mathcal{T}} B \cdot dl \]

**Region-based Distinction**

Pseudo-Homology:
\[ \mathcal{H}(T' + T'_h) = \mathcal{H}(T'' + T''_h) \]

Helper trajectory for corridor-like region:
Proposed Method III: Sampling-based Maneuver Pattern Analysis

**Motivation:** factors other than topology that matter for pattern distinction of spatiotemporal trajectory?

**Conservative**  **Aggressive**

**Pseudo-Homological**

**Overtaking Sequence-based Distinction**
- Forward simulate APV & objects.
- Keep track of the overtaking timestamp.
- Sort obstacles (identifier) based on the overtaking timestamp.

**Maneuver Pattern Distinction Tree**
Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR
Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR

- **Iterative LQR Backgrounds**

  Smoothened model-feasible path/trajectory
  - Does not consider any costs other than tracking errors, e.g., obstacles.
  - Smoothed trajectory is only model-feasible, but not execution-feasible.

  **Trajectory Representation & Cost**

  \[ x_{i+1} = f_d(x_i, u_i) \]
  \[ X^{(k)} = \{x_0, x_1, \ldots, x_i, \ldots, x_{N-1}, x_N \} \]
  \[ U^{(k)} = \{u_0, u_1, \ldots, u_i, \ldots, u_{N-1} \} \]

  **Iterative Linear Quadratic Regulator**
  (1st order Differential Dynamic Programming)

  \[ \delta x \text{ incurred at (i-1)}^{th} \text{ timestamp.} \]
  \[ \delta u \text{ to be determined at (i)}^{th} \text{ timestamp.} \]

  **Perturbation:**

  \[ \delta u^* = \arg \min_{\delta u} \tilde{Q}(\delta x, \delta u) \]

  \[ = k + K \cdot \delta x \]

  **Singular Value Decomposition**

  \[ Q_{uu} = P \Sigma Q^T \]
  \[ Q_{uu}^{-1} = Q \Sigma^{-1} P^T \]

  \[ k = -Q_{uu}^{-1} Q_u \]
  \[ K = -Q_{uu}^{-1} Q_{ux} \]

  \[ \Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \ddots & \sigma_n \end{bmatrix} \]
  \[ \Sigma^{-1} = \begin{bmatrix} \frac{1}{\sigma_1 + \lambda} & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \ddots & \frac{1}{\sigma_n + \lambda} \end{bmatrix} \]

  \[ + \lambda \]
Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR

**Focused Iterative-LQR & Cost Function**

Determine $\lambda$

### Levenberg-Marquardt Heuristics

- **iLQR Optimization making progress:** $\lambda$ ❯
- **iLQR Optimization not making progress:** $\lambda$ ➡

### Focused Line-Search Heuristics

- **Optimization violating maneuver pattern constraint:** $\lambda$ ➡
- **iLQR Optimization making progress:** $\lambda$ ❯
- **Line-Search Optimization making progress:** $\lambda$ ↔
- **iLQR Optimization not making progress:** $\lambda$ ➡

Cost Function

Cost penalizes certain undesirable aspect of the plan/trajectory.

**Total Cost:**

$$J(X, U) = g_N(x_N) + \sum_{i=0}^{N-1} g(x_i, u_i)$$

- **Cost terms:**
  - $g(x_i, u_i) = \sum_{k=1}^{M} \omega_k c_k(x_i, u_i)$
  - $g_N(x_N) = \sum_{k=1}^{M} \omega_k c_k(x_N, 0)$

**Weights Cost Terms**

Better be Continuous?

- **Edge-Augmented Graph Search** ✗
- **Topological Region/Pattern Selection** ✗
- **Iterative-LQR Trajectory Optimization** ✓
Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR

- **Cost Function Design**

  iLQR is underlying Newton’s Method:
  - Convex Optimization
  - 2nd-order Continuous

  **Key Results in Convex Optimization**
  - 2nd-order continuous, monotonically non-decreasing convex function $h(y)$
  - 2nd-order continuous, differentiable convex $g(x)$

  $f = (h \circ g)(x) = h(g(x))$

  **Still Convex!**

  **Quadratic**

  **State/Control Variable**
  
  E.g., steering, speed, swirl, acceleration, etc.

  **Distance Function**
  
  - Disk
  - Polygon
  - Polyline

  **Convex Modulation Function**

  **Convex Feature Function**

  **Total Cost Function**

  **Behavioral Feature**

  $$ g = e_{\text{obstacle}}^h(\omega_{\text{dist}}) + e_{\text{pol}}^h(\omega_{\text{pol}}) + e_{\text{speed}}^h(\omega_{\text{speed}}) + e_{\text{lat-acc}}^h(\omega_{\text{lat-acc}}) + e_{\text{swirl}}^h(\omega_{\text{swirl}}) + e_{\text{lat-acc}}^h(\omega_{\text{lat-acc}}) $$

  **Constraint Feature:**
  - Penalty Method
  - Constrained Iterative-LQR Optimization
  - Execution Feasible Trajectory

  **Non-convex Features**

  **Penetrated Distance:**

  **Distance** → **Penetrated Distance** → **Non-Convex!**

  **Local cost quadratization → Topological Structure**
Results

- Planning Framework / Algorithmic Flow
Results

• Experiment Setup

Scenario:
A particular setup of the environment elements including lanes, obstacles and the APV.

Snapshot:
Plots of the environment elements’ states and the planning outcome of each planning module at a given time-stamp.

Overlay:
A plot of the overlaid states of the environment elements’ states over a time period that the APV demonstrates a maneuver.
Results

- Simulation Scenario 1
Results

- Simulation Scenario 2
Results

- Simulation Scenario 3
Results

- Simulation Scenario 4
Results

- Simulation Scenario 5
Results

- On-Vehicle (Simulation)
Results

• On-Vehicle (Closed Course)

Autonomous Driving Test (On-Vehicle)

Avoid Tightly Spaced Obstacles
Results

- On-Vehicle (Closed Course)
Results

• On-Vehicle (Schenley Park)
Results

- Past demo video footages.

[33-mile Autonomous Drive from Cranberry, Pennsylvania to Pittsburgh International Airport](Link)

[Highway Driving Near Capitol Hill, Washington DC I-395 South Multiple lane merges & changes](Link)
## Conclusions

- **Comparison**

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Conclusions

• Summary

**Requirement 1: a deliberative trajectory planning system.**
Reference and Trajectory Planning have long spatial (100m) or temporal horizon (6s).

**Requirement 2: spatiotemporal (trajectory) planning.**
Maneuver pattern analysis and focused trajectory optimization explicitly plan spatiotemporally.

**Requirement 3: tactical reasoning capability with topological awareness.**
Region segmentation and maneuver pattern analysis provides tactical reasoning capability.

**Requirement 4: global awareness with the ability to converge to a local optimum.**
Reference planning and maneuver pattern analysis both perform global sampling.

**Requirement 5: apply to self-driving passenger vehicle on-road driving.**
Extensively experimented on a real self-driving vehicle.
Conclusions

- Contributions

  1. A Hybrid Trajectory Planning Framework
  2. Search over Edge-augmented Graph for Reference Path Planning
  3. Topological Backward Propagation Algorithm for Region (Sub-Graph) Segmentation
  4. Sampling-based Maneuver Pattern Analysis/Seeding Trajectory Generation for On-Road Self-Driving
  5. Adapted Linear Quadratic Regulator (LQR) and Iterative-LQR for Trajectory Smoothing/Optimization
  6. Identification of Useful Cost Function Generation Principles
Future Work

- TBC: the fusion of sampling-based and optimization-based method.
- TBC: The inclusion in topological analysis in a trajectory planning system.
- Automated parameter tuning through machine learning techniques.
- Unstructured learning of driving skill, a.k.a, neural network, etc.
- Misc:
  - More complex/realistic vehicle model.
  - Planning with better shape representations.
  - Faster collision checking (distance function evaluation).
  - Faster homology information calculation.
Publications

- Gu, T., et al. (2013). Focused trajectory planning for autonomous on-road driving. Intelligent Vehicles Symposium (IV), 2013, IEEE.
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